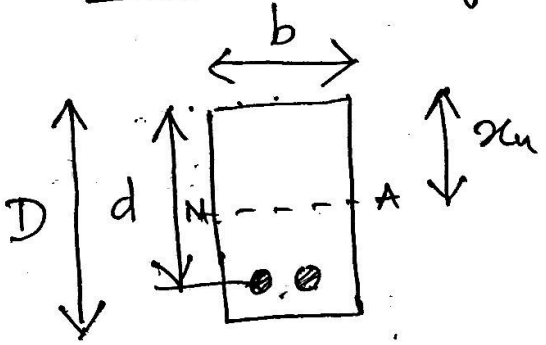
Design of Concrete Structures (First, Singly Reinforced Beams)



$C = 0.36 f_{ck} b x_u$

$T = 0.87 f_y A_{st}$ ← under-reinforced & balanced section

for over-reinf s/n,

$T = f_{st} A_{st}$

$C = T$

$0.36 f_{ck} b x_u = 0.87 f_y A_{st}$

$\Rightarrow x_u = \dfrac{0.87 f_y A_{st}}{0.36 f_{ck} b}$ ✓

← general eqn

for balanced section,

$x_{umax} = 0.48d$ (Fe 415)

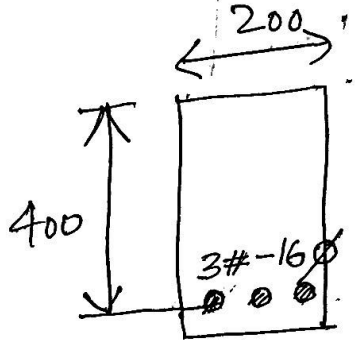
$= 0.46d$ (Fe 500)

$= 0.53d$ (Fe 250)

If $x_u < x_{umax} \Rightarrow$ Under-reinforced

$x_u = x_{umax} \Rightarrow$ Balanced ✓

Q: Find moment of resistance of a singly reinforced beam 200 mm wide & 400 mm effective depth, reinforced with 3 bars of 16 mm dia of Fe415 grade steel. Use M20 concrete.



Ans:

To find x_u,

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 415 \times 3\pi/4 \times 16^2}{0.36 \times 20 \times 200}$$

$$= 151.24 \text{ mm} \checkmark$$

$f_y = 415 \text{ N/mm}^2$

$f_{ck} = 20 \text{ N/mm}^2$

$b = 200 \text{ mm}$

$A_{st} = 3 \times \left(\frac{\pi}{4} \times 16^2\right) \text{ mm}^2$

$x_{umax} = 0.48 d$ (For Fe415 steel)

$= 0.48 \times 400$

$= 192 \text{ mm}$

Since $x_u < x_{umax} \Rightarrow$ Under-reinforced Beam

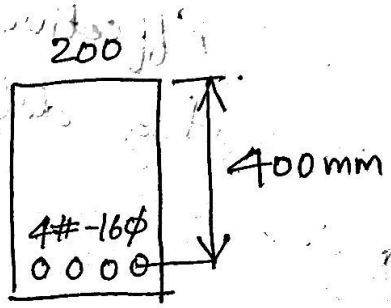
Moment of resistance:

$M_u = C \cdot z = 0.36 f_{ck} b x_u (d - 0.42 x_u)$

$= 0.36 \times 20 \times 200 \times 151.24 (400 - (0.42 \times 151.24))$

$= 73.28 \times 10^6 \text{ Nmm} = 73.28 \text{ kNm}$

Q: Determine the moment of resistance of a beam section 200 mm wide and 400mm effective depth, with 4 bars of 16mmϕ of Fe250 steel. Use M20 concrete.



Ans:

Depth of neutral axis

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 250 \times 4\pi/4 \times 16^2}{0.36 \times 20 \times 200}$$

$$= 121.50 \text{ mm}$$

$f_y = 250 \text{ N/mm}^2$

$f_{ck} = 20 \text{ N/mm}^2$

$b = 200 \text{ mm}$

$A_{st} = 4(\pi/4 \times 16^2) \text{ mm}^2$

$x_{umax} = 0.53 d$ (for fe250)

$= 0.53 \times 400 = 212 \text{ mm}$

Since $x_u < x_{umax}$ $\Rightarrow$ Under reinforced section

Moment of Resistance

$$M_u = C.z = 0.36 f_{ck} b x_u (d - 0.42 x_u)$$

$$= 0.36 \times 20 \times 200 \times 121.50 (400 - 0.42 \times 121.50)$$

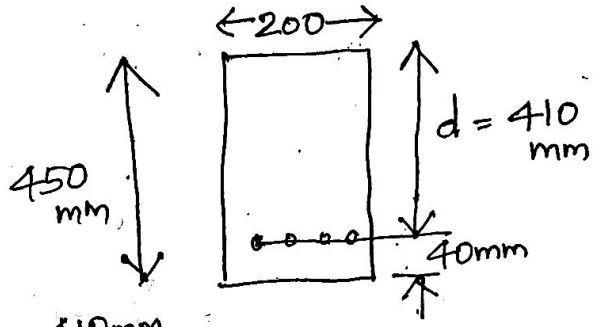
$$= 61.06 \times 10^6 \text{ Nmm}$$

$$= 61.06 \text{ kNm}$$

Q: Find limiting moment of resistance of a singly reinforced beam of size 200 x 450 mm. Use M25 concrete & fe415 grade steel. Effective cover to reinforcement is 40 mm. Also determine limiting percentage of steel.

Ans:

To find, $M_{u,lim}$ & $\%A_{st,lim}$ ✓ ✓



From fig, effective depth $d = 450 - 40 = \underline{410 mm}$

$f_{ck} = 25 N/mm^2$

$f_y = 415 N/mm^2$

For balanced section,

$x_u = x_{umax}$

$= 0.48d$ (for fe415)

$= 0.48 \times 410 = 196.80 mm$

Here, limiting moment of resistance

$M_{u,lim} = 0.36 f_{ck} b x_u (d - 0.42 x_u)$

$= 0.36 \times 25 \times 200 \times 196.80 (410 - 0.42 \times 196.80)$

$= 115.96 \times 10^6 Nmm = \underline{115.96 kNm}$

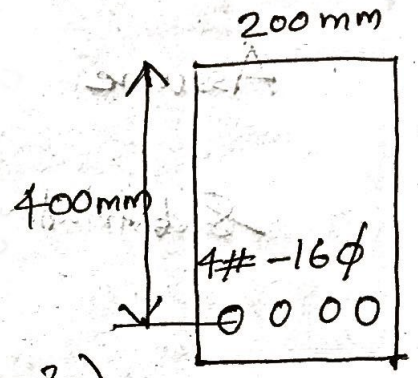
We know $x_u = 196.80 mm$

$\Rightarrow \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = 196.80.$

$\Rightarrow A_{st(lim)} = \frac{196.80 \times 0.36 \times 25 \times 200}{0.87 \times 415} = \underline{981.14 mm^2}$

Limiting % steel $= \frac{A_{st,lim}}{bd} \times 100 = \frac{981.14 \times 100}{200 \times 410} = \underline{1.197\%}$

Q: Find moment of resistance of a singly reinforced RCC beam of 200mm width and 400 mm effective depth, reinforced with 4 bars of 16 mm ϕ of fe415 steel. Use M20 concrete.



Ans: Depth of neutral axis

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 415 \times 4(\pi/4 \times 16^2)}{0.36 \times 20 \times 200}$$

$$= 201.70 \text{ mm}$$

$f_y = 415 \text{ N/mm}^2$

$f_{ck} = 20 \text{ N/mm}^2$

$b = 200 \text{ mm}$

$A_{st} = 4(\pi/4 \times 16^2) \text{ mm}^2$

$x_{umax} = 0.48 d$ (for fe415)

$= 0.48 \times 400 = 192 \text{ mm}^2$

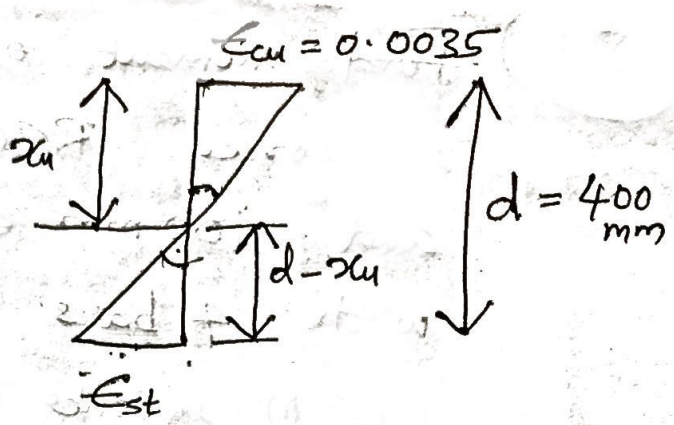
Since $x_u > x_{umax}$ $\Rightarrow$ Over-reinforced section

Since for over-reinf section, steel may not have yielded. So stress in steel is less than $0.87 f_y$.

ϵ_{st} ? $\rightarrow$ f_{st} ?

$$\frac{0.0035}{x_u} = \frac{\epsilon_{st}}{d - x_u}$$

$$\Rightarrow \epsilon_{st} = 0.0035\left(\frac{d - x_u}{x_u}\right)$$



Assume $x_u = \frac{201.70 + 192}{2} = 197$ mm

Substituting, we get, $\epsilon_{st} = 0.0035\left(\frac{400 - 197}{197}\right)$

$= 0.0036$

$f_{st} = 355\ N/mm^2$

$$x_u = \frac{f_{st}\ A_{st}}{0.36\ f_{ck}\ b} = \frac{355 \times 4\pi/4 \times 16^2}{0.36 \times 20 \times 200}$$

$= 198$ mm

Final value of $x_u = \frac{197 + 198}{2} = 197.50$ mm

$$M_u = 0.36 f_{ck}\ b\ x_u (d - 0.42 x_u)$$

$= 0.36 \times 20 \times 200 \times 197.50 (400 - 0.42 \times 197.50)$

$= 90 \times 10^6$ Nmm

$= 90$ kNm